

## Flow and Heat Transfer of a Power-Law Fluid over a Permeable Shrinking Sheet (Aliran dan Pemindahan Haba bagi Bendalir Hukum-Kuasa di Atas Lembaran Telap yang Mengecut)

NOR AZIZAH YACOB & ANUAR ISHAK\*

### ABSTRACT

*The steady, two-dimensional laminar flow of a power-law fluid over a permeable shrinking sheet of constant surface temperature is investigated. The governing partial differential equations were transformed into a system of nonlinear ordinary differential equations using a similarity transformation, before being solved numerically by the Runge-Kutta-Fehlberg method with shooting technique. The results are presented graphically and the effects of the power-law index  $n$ , suction parameter  $f_w$  and Prandtl number  $Pr$  were discussed. It was found that stronger suction is necessary for the solution to exist for a pseudoplastic fluid ( $n < 1$ ) compared to a dilatant fluid ( $n > 1$ ).*

*Keywords: Boundary layer; heat transfer; power-law fluid; shrinking sheet*

### ABSTRAK

*Aliran lamina mantap dua matra bendalir hukum-kuasa di atas lembaran telap dengan suhu permukaan malar yang mengecut dikaji. Persamaan pembezaan separa dijelmakan menjadi satu sistem persamaan pembezaan biasa tak linear menggunakan penjelmaan keserupaan, sebelum diselesaikan secara berangka menggunakan kaedah Runge-Kutta-Fehlberg dengan teknik tembakan. Keputusan dibentangkan secara grafik dan kesan indeks hukum-kuasa  $n$ , parameter sedutan  $f_w$  dan nombor Prandtl  $Pr$  dibincangkan. Didapati bahawa sedutan yang kuat adalah perlu supaya penyelesaian wujud bagi bendalir pseudoplastik ( $n < 1$ ) berbanding dengan bendalir dilatan ( $n > 1$ ).*

*Kata kunci: Bendalir hukum-kuasa; lapisan sempadan; lembaran mengecut; pemindahan haba*

### INTRODUCTION

The study of a non-Newtonian fluid flow and heat transfer over a stretching surface has received great attention due to its numerous industrial applications. There are many industrially important fluids that exhibit non-Newtonian fluid behavior such as polymer solutions, molten plastics, pulps and foods (Chen 2008). Acrivos et al. (1960) and Schowalter (1960) probably were the first who have investigated the boundary layer flow of a non-Newtonian power-law fluid. Thereafter, many researchers have investigated the flow and heat transfer of this kind of fluid such as Hassanien (1996), who has investigated the steady flow and heat transfer characteristics of a continuous flat surface moving in a parallel free stream using the finite difference approximations, while Howell et al. (1997) studied the similar problem in a quiescent fluid where the free stream velocity is zero by using the Merk-Chao series expansion method. Further, Chen (2008) studied the effects of a magnetic field and suction/injection on the flow and heat transfer over a stretched sheet and found that the presence of a magnetic field increases the surface friction but decreases the rate of heat transfer from the sheet. On the other hand, the imposition of suction is to increase both the skin friction coefficient and the heat transfer rate at the surface, whereas injection shows an opposite effect.

Zheng et al. (2008) presented the theoretical analysis for the boundary layer flow over a continuously moving

surface in an otherwise quiescent pseudo-plastic fluids. Cheng and Liu (2009) have employed a long-wave perturbation method to analyze the stability of an electrically-conductive power-law fluid traveling down a vertical cylinder in a magnetic field. They found that the stability of the film flow system is weakened as the radius of the cylinder is reduced. However, the flow stability can be enhanced by increasing the intensity of the magnetic field and the flow index, respectively. Prasad et al. (2010) considered the effect of thermal buoyancy on the flow past a vertical continuous stretching sheet in the presence of a magnetic field. The magnetohydrodynamic flow of a power-law fluid over a stretching sheet was considered by Cortell (2005), while Mahapatra et al. (2009) reported the analytical solution of magnetohydrodynamic stagnation-point flow of a power-law fluid towards a stretching surface. Very recently Postelnicu and Pop (2011) studied the Falkner-Skan boundary layer flow of a power-law fluid past a stretching wedge and reported the existence of dual solutions when the wedge stretches against the mainstream. Different from the above studies, Miklavčič and Wang (2006) investigated the flow due to a shrinking sheet where the velocity boundary layer moves toward a fixed point. They found an exact solution of the Navier-Stokes equations and reported that vorticity of the shrinking sheet is not confined within a boundary layer and the flow is unlikely to exist unless adequate suction on the boundary

is imposed. Later, Fang et al. (2008) reported a theoretical estimation of the solution domain and solved the Blasius equation with the associated boundary conditions using a numerical technique. Further, Fang et al. (2010) solved analytically the slip flow over a permeable shrinking surface in a viscous fluid, which is modeled using the newly proposed second order slip flow. On the other hand, Fang and Zhang (2009) and Hayat et al. (2007) have included the magnetohydrodynamic (MHD) effect on the steady shrinking problems in viscous fluid and solved the problems analytically.

The shrinking sheet problem was also extended to a power-law velocity as reported by Fang (2008). Fang and Zhang (2010) successfully obtained the analytical solution for the thermal boundary layer over a linearly shrinking sheet. Ishak et al. (2010) considered the stagnation-point flow over a shrinking sheet immersed in a micropolar fluid, Bachok et al. (2010) solved the problem of unsteady three-dimensional boundary layer flow due to a permeable shrinking sheet, and Arifin et al. (2010) considered the flow over a permeable stretching/shrinking sheet in a nanofluid. Recently, Yao et al. (2011) investigated the heat transfer of a viscous fluid flow over a stretching/shrinking sheet with a convective boundary condition and reported the solution in an incomplete Gamma function.

However, the above mentioned studies were restricted to the problems of a shrinking sheet immersed in a viscous fluid. The laminar boundary layer flow and heat transfer of a non-Newtonian power-law fluid over a stretching sheet was investigated by Xu and Liao (2009). They considered the stretching velocity of the form  $U_w(x) = ax^m$  and found that the similarity solutions are in existence for both the momentum and energy equations when  $m = 1/3$ . Different from those considered by Xu and Liao (2009), the present study considers the case of a shrinking sheet and show that the solutions are not unique, and exist only if adequate suction on the boundary is imposed.

MATHEMATICAL MODEL

Consider a steady, two-dimensional laminar flow of a power-law fluid over a permeable shrinking sheet of constant surface temperature  $T_w$  as shown in Figure 1. The shrinking velocity is assumed to be of the form  $U_w(x) = ax^{1/3}$ , where  $a$  is a positive constant. The  $x$ -axis extends parallel, while the  $y$ -axis extends upwards, normal to the surface of the sheet. The boundary layer equations are (Howell et al. 1997; Wang 1994; Xu & Liao 2009):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial \tau_{xy}}{\partial y}, \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}, \tag{3}$$

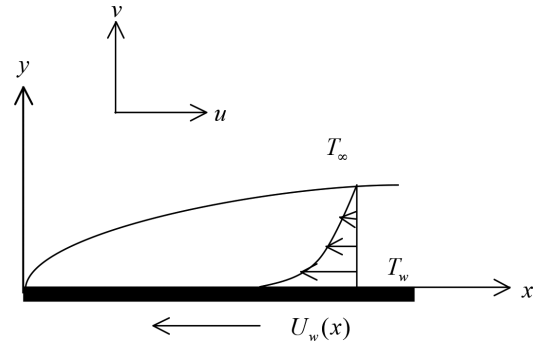


FIGURE 1. Physical model and coordinate system

subject to the boundary conditions,

$$\begin{aligned} u &= -U_w(x), \quad v = V_w(x), \quad T = T_w \quad \text{at } y = 0, \\ u &\rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty, \end{aligned} \tag{4}$$

where  $u$  and  $v$  are the velocity components along the  $x$  and  $y$  directions, respectively,  $\tau_{xy}$  is the shear stress,  $\rho$  is the fluid density and  $V_w(x)$  (which will be defined later) is the mass transfer velocity at the surface of the sheet.

The stress tensor is defined as (Andersson & Irgens 1990; Wilkinson 1960),

$$\tau_{ij} = 2K(2D_{ij}D_{kl})^{(n-1)/2}D_{ij}, \tag{5}$$

where

$$D_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \tag{6}$$

denotes the stretching tensor,  $K$  is called the consistency coefficient and  $n$  is the power-law index. The index  $n$  is non-dimensional and the dimension of  $K$  depends on the value of  $n$ . The two-parameter rheological (5) is known as the Ostwald-de-Waele model or, more commonly, the power-law model. The parameter  $n$  is an important index to subdivide fluids into pseudoplastic fluids ( $n < 1$ ) and dilatant fluids ( $n > 1$ ). For  $n = 1$ , the fluid is simply the Newtonian fluid. Therefore, the deviation of  $n$  from unity indicates the degree of deviation from Newtonian behavior (Wang 1994). With  $n \neq 1$ , the constitutive (5) represents shear-thinning ( $n < 1$ ) and shear-thickening ( $n > 1$ ) fluids. Using (5) and (6), the shear stress appearing in (2) can be written as:

$$\tau_{xy} = K \left( \frac{\partial u}{\partial y} \right)^n. \tag{7}$$

Now the momentum (2) becomes:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{K}{\rho} \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right)^n. \tag{8}$$

The continuity (1) is satisfied by introducing a stream function  $\psi$  such that  $u = \partial\psi/\partial y$  and  $v = -\partial\psi/\partial x$ . The

momentum (8) and the energy (3) can be transformed into the corresponding ordinary differential equations by the following transformation (Xu & Liao 2009):

$$\eta = \frac{y}{x} (Re_x)^{1/(n+1)}, \quad \psi = U_w x (Re_x)^{-1/(n+1)} f(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad (9)$$

where  $\eta$  is the similarity variable,  $f(\eta)$  is the dimensionless stream function and  $Re_x = \rho x^n U_w^{2-n}/K$  is the local Reynolds number. Using transformation (9), we obtain,

$$u = ax^{1/3} f'(\eta), \quad v = -\frac{1}{3} a \left( \frac{\rho a^{2-n}}{K} \right)^{\frac{1}{n+1}} x^{-1/3} (2f - \eta f'), \quad (10)$$

and thus to obtain similarity, the mass transfer velocity  $V_w(x)$  may be defined as:

$$V_w(x) = -\frac{2}{3} a \left( \frac{\rho a^{2-n}}{K} \right)^{\frac{1}{n+1}} x^{-1/3} f_w, \quad (11)$$

where  $f_w = f(0)$  is the suction/injection parameter with  $f_w > 0$  is for suction,  $f_w < 0$  is for injection and  $f_w = 0$  corresponds to an impermeable plate. The transformed nonlinear ordinary differential equations are:

$$n(f'')^{n-1} f''' + \frac{2}{3} f f'' - \frac{1}{3} f'^2 = 0, \quad (12)$$

$$\frac{1}{Pr} \theta'' + \frac{2}{3} f \theta' = 0, \quad (13)$$

where primes denote differentiation with respect to  $\eta$ , and

$Pr = \frac{a}{\alpha} \left( \frac{\rho a^{2-n}}{K} \right)^{\frac{2}{n+1}}$  is the generalized Prandtl number.

The transformed boundary conditions are:

$$f(0) = f_w, \quad f'(0) = -1, \quad \theta(0) = 1,$$

$$f'(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty. \quad (14)$$

The physical quantities of interest are the skin friction coefficient  $C_f$  and the local Nusselt number  $Nu_x$ , which are defined as:

$$C_f = \frac{\tau_w}{\rho U_w^2 / 2}, \quad Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, \quad (15)$$

where the surface shear stress  $\tau_w$  and the surface heat flux  $q_w$  are given:

$$\tau_w = \left[ K \left( \frac{\partial u}{\partial y} \right)^n \right]_{y=0}, \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}, \quad (16)$$

with  $\mu$  and  $k$  being the dynamic viscosity and the thermal conductivity, respectively. Using the similarity variables (9), we obtain:

$$\frac{1}{2} C_f Re_x^{1/(n+1)} = [f''(0)]^n, \quad Nu_x Re_x^{-1/(n+1)} = -\theta'(0). \quad (17)$$

## NUMERICAL METHOD

The nonlinear differential (12) and (13) along with the boundary conditions (14) form a two-point boundary value problem (BVP) and are solved using a shooting method, by converting them into an initial value problem (IVP). This method is very well described in the recent papers by Bhattacharyya and Layek (2011) and Bhattacharyya et al. (2011). In this method, we choose suitable finite values of  $\eta$ , say  $\eta_\infty$ , which depend on the values of the parameters considered. First, the system of (12) and (13) is reduced to a first-order system (by introducing new variables) as follows:

$$f' = p, \quad \dot{p} = q, \quad nq^{n-1}q' + \frac{2}{3}fq - \frac{1}{3}p^2 = 0, \quad (18)$$

$$\theta' = r, \quad \frac{1}{Pr}r' + \frac{2}{3}fr = 0, \quad (19)$$

with the boundary conditions,

$$f(0) = f_w, \quad p(0) = -1, \quad \theta(0) = 1,$$

$$p(\eta_\infty) = 0, \quad \theta(\eta_\infty) = 0. \quad (20)$$

Now we have a set of 'partial' initial conditions,

$$f(0) = f_w, \quad p(0) = -1, \quad q(0) = ?, \quad \theta(0) = 1, \quad r(0) = ?. \quad (21)$$

As we notice, we do not have the values of  $q(0)$  and  $r(0)$ . To solve (18) and (19) as an IVP, we need the values of  $q(0)$  and  $r(0)$ , i.e.  $f''(0)$  and  $\theta'(0)$ . We guess these values, and apply the Runge-Kutta-Fehlberg method, then see if this guess matches the boundary conditions at the very end. Varying the initial slopes give rise to a set of profiles which suggest the trajectory of a projectile 'shot' from the initial point. That initial slope is sought which results in the trajectory 'hitting' the target, that is, the final value (Bailey et al. 1968).

To determine either the solution obtained is valid or not, it is necessary to check the velocity and the temperature profiles. The correct profiles must satisfy the boundary conditions at  $\eta = \eta_\infty$  asymptotically. This procedure is repeated for other guessing values of  $q(0)$  and  $r(0)$  for the same values of parameters. If a different solution is obtained and the profiles satisfy the far field boundary conditions asymptotically but with different boundary layer thickness, then this solution is also a solution to the boundary-value problem (second solution).

## RESULTS AND DISCUSSION

The numerical computations were carried out for various values of parameters involved, namely the power-law index  $n$ , suction parameter  $f_w$  and Prandtl number  $Pr$ . The data obtained are tabulated and then used to sketch the graphs as shown in Figures 2 and 3.

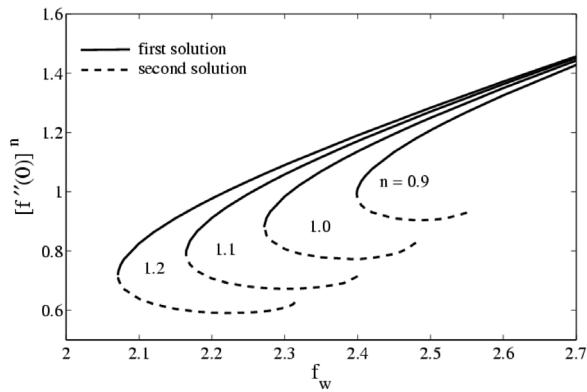


FIGURE 2. Variation of  $[f''(0)]^n$  with  $f_w$  for different values of  $n$

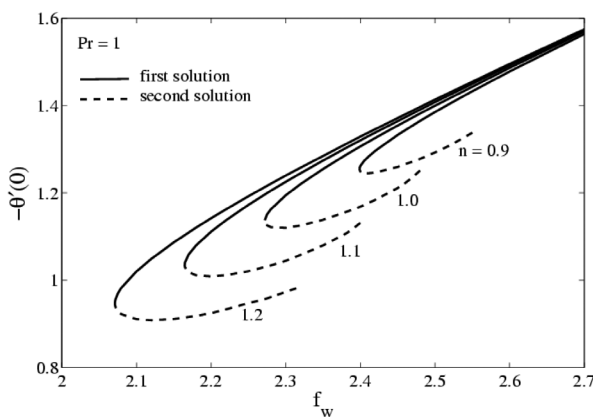


FIGURE 3. Variation of  $-\theta'(0)$  with  $f_w$  for different values of  $n$  when  $Pr = 1$

Figure 2 presents the variation of the skin friction coefficient  $[f''(0)]^n$  with  $f_w$  for different values of  $n$ , while the respective local Nusselt number  $-\theta'(0)$  is displayed in Figure 3. These figures show that, an adequate suction  $f_w$  is needed for the solution to exist, which is in agreement with the results reported by Fang et al. (2008) and Miklavčič and Wang (2006). In contrast to the stretching sheet problem, the solutions are not unique, where dual solutions are found to exist for a certain range of  $f_w$ . Figures 2 and 3 show that increasing  $n$  is to widen the range of  $f_w$  for which the solution exists. Table 1 shows the critical values  $f_c$  for different values of  $n$  when  $Pr = 1$ . It can be seen that the value  $f_c$  decreases with an increase in  $n$  as presented in Figures 2 and 3. There is no solution obtained beyond this critical value, whereas dual solution exist when  $f_w > f_c$  for a certain range of  $f_w$ . The curve for a particular value of  $n$  bifurcates at  $f_w = f_c$  and the second solution continues further and terminates at a certain value of  $f_w$ . It should be remarked that the computations have been performed until the point where the solution does not converge, and the calculations were terminated at that point. Moreover, from Figures 2 and 3 it is seen that stronger suction is necessary for the solution to exist for a pseudoplastic fluid ( $n < 1$ ) compared to a dilatant fluid ( $n > 1$ ). Table 2 shows the values of the skin friction

TABLE 1. Critical values  $f_c$  for which the solution exists, for different values of  $n$

$n$	$f_c$
0.9	2.39879
1	2.27204
1.1	2.16466
1.2	2.07120

TABLE 2. Values of the skin friction coefficient  $[f''(0)]^n$  and the local Nusselt number  $-\theta'(0)$  for the first solutions for different values of  $n$  and  $f_w$  when  $Pr = 1$

$n$	$f_w$	$[f''(0)]^n$	$-\theta'(0)$
0.6	3.3	1.9198	2.0307
0.7	3.0	1.6783	1.7989
0.9	2.5	1.2070	1.3851
1.0	2.3	0.9840	1.1946
1.1	2.2	0.9116	1.1112
1.2	2.1	0.8253	1.0203
1.3	2.0	0.7204	0.9156
1.4	1.93	0.6780	0.8594

coefficient  $[f''(0)]^n$  and the local Nusselt number  $-\theta'(0)$  for the first solutions for different values of  $n$  and  $f_w$ . It can be seen that the skin friction coefficient and the local Nusselt number which represents the heat transfer rate at the surface is lower for the dilatant fluid ( $n > 1$ ) compared to the pseudoplastic fluid ( $n < 1$ ). As discussed by Miklavčič and Wang (2006) for the classical viscous fluid ( $n = 1$ ), stretching sheet solution would induce a far field suction towards the sheet, while the shrinking sheet would cause a velocity away from the sheet. Thus from physical grounds, vorticity of the shrinking sheet is not confined within a boundary layer and the flow is unlikely to exist unless adequate suction on the boundary is imposed. Thus, the solution exists only for the case of suction ( $f_w > 0$ ). Moreover, Fang and Zhang (2010) showed that for a classical viscous fluid ( $n = 1$ ), there are two solutions for  $f_w > 2$ , one solution for  $f_w = 2$  and no solution exists for  $f_w < 2$ . Based on our computations, it is possible to get solution for  $n \rightarrow 0$ , but the velocity and temperature profiles do not reach the far field boundary conditions asymptotically for  $n > 1.4$ .

The velocity and the thermal boundary layer thicknesses decrease with  $n$  for the first solution as presented in Figures 4 and 5. The decreasing of the velocity and thermal boundary layer thicknesses imply the increasing manner of the velocity and temperature gradients at the surface, and in consequence increase the skin friction coefficient and the local Nusselt number (in absolute sense) as displayed in Figures 2 and 3. The opposite behaviors are observed for the second solutions. Figure 6 shows the temperature profiles for different values of  $Pr$  when  $n = 1.1$  and  $f_w = 2.3$ . As can be seen, the thermal boundary layer thickness decreases with  $Pr$  for both first and second solutions and consequently

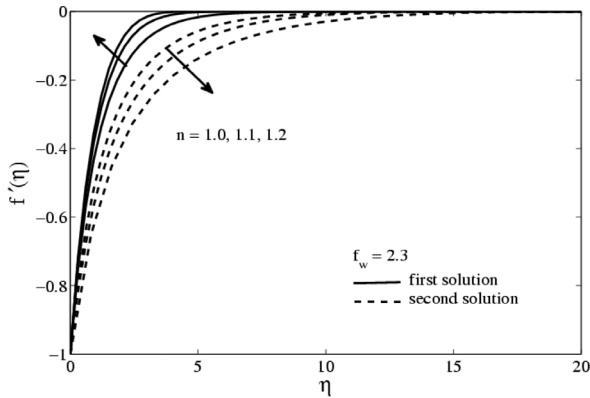


FIGURE 4. Velocity profiles  $f'(\eta)$  for different values of  $n$  when  $f_w = 2.3$

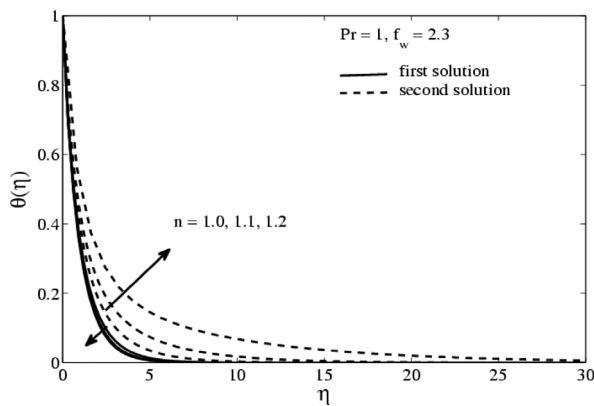


FIGURE 5. Temperature profiles  $\theta(\eta)$  for different values of  $n$  when  $Pr = 1$  and  $f_w = 2.3$

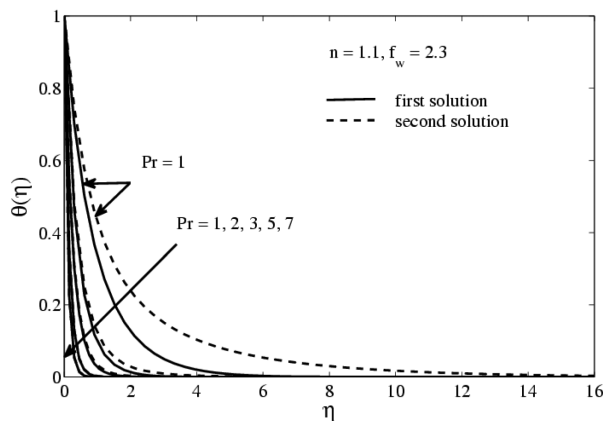


FIGURE 6. Temperature profiles  $\theta(\eta)$  for different values of  $Pr$  when  $n = 1.1$  and  $f_w = 2.3$

increases the local Nusselt number. It is worth mentioning that the nonlinear ordinary differential (12) and (13) are uncoupled and thus Prandtl number  $Pr$  has no influence to the flow field. Figures 4–6 show that the far field boundary conditions (14) are satisfied asymptotically, thus supports the validity of the numerical results

obtained, besides supporting the dual nature of the solutions presented in Figures 2 and 3. Between the two solutions, which solution is stable and most physically relevant depends on the stability of the solution, which is not considered here. However, we suggest that the first solution is stable and the second solution is not, since the first solution is the only solution for large values of the suction parameter.

## CONCLUSION

The problem of a steady, two-dimensional laminar flow of a power-law fluid over a permeable shrinking sheet of constant surface temperature was investigated numerically. The governing partial differential equations were transformed into a system of nonlinear ordinary differential equations using a similarity transformation, before being solved numerically by the Runge-Kutta-Fehlberg method with shooting technique. The results were presented graphically and the effects of the power-law index  $n$ , suction parameter  $f_w$  and Prandtl number  $Pr$  were discussed. It was found that the heat transfer rate at the surface increases with an increase in the Prandtl number. Moreover, stronger suction is necessary for the solution to exist for a pseudoplastic fluid ( $n < 1$ ) compared to a dilatant fluid ( $n > 1$ ).

## ACKNOWLEDGEMENTS

The financial supports received in the form of research grants from the Ministry of Higher Education, Malaysia (Project Code: FRGS/1/2012/SG04/UKM/01/1) and the Universiti Kebangsaan Malaysia (Project Code: UKM-GUP-2011-202) are gratefully acknowledged.

## REFERENCES

- Acrivos, A., Shah, M.J. & Petersen, E.E. 1960. Momentum and heat transfer in laminar boundary-layer flows of non-Newtonian fluids past external surfaces. *AIChE Journal* 6: 312-317.
- Andersson, H.I. & Irgens, F. 1990. Film flow of power-law fluids. In *Encyclopedia of Fluid Mechanics*, edited by Chermisinoff, N.P. Houston: Gulf Publishing, pp.617-648.
- Arifin, N.M., Nazar, R. & Pop, I. 2010. Viscous flow due to a permeable stretching/shrinking sheet in a nanofluid. *Sains Malaysiana* 40: 1359-1367.
- Bachok, N., Ishak, A. & Pop, I. 2010. Unsteady three-dimensional boundary layer flow due to a permeable shrinking sheet. *Applied Mathematics and Mechanics - English Edition* 31: 1421-1428.
- Bailey, P.B., Shampine, L.F. & Waltman, P.E. 1968. *Nonlinear Two Point Boundary Value Problems*. New York: Academic Press.
- Bhattacharyya, K. & Layek, G.C. 2011. Effects of suction/blowing on steady boundary layer stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation. *International Journal of Heat and Mass Transfer* 54: 302-307.
- Bhattacharyya, K., Mukhopadhyay, S. & Layek, G.C. 2011. Slip effects on boundary layer stagnation-point flow and heat

- transfer towards a shrinking sheet. *International Journal of Heat and Mass Transfer* 54: 308-313.
- Chen, C.H. 2008. Effects of magnetic field and suction/injection on convection heat transfer of non-Newtonian power-law fluids past a power-law stretched sheet with surface heat flux. *International Journal of Thermal Sciences* 47: 954-961.
- Cheng, P.-J. & Liu, K.-C. 2009. Hydromagnetic instability of a power-law liquid film flowing down a vertical cylinder using numerical approximation approach techniques. *Applied Mathematical Modelling* 33: 1904-1914.
- Cortell, R. 2005. A note on magnetohydrodynamic flow of a power-law fluid over a stretching sheet. *Applied Mathematics and Computation* 168: 557-566.
- Fang, T. 2008. Boundary layer flow over a shrinking sheet with power-law velocity. *International Journal of Heat and Mass Transfer* 51: 5838-5843.
- Fang, T. & Zhang, J. 2010. Thermal boundary layers over a shrinking sheet: An analytical solution. *Acta Mechanica* 209: 325-343.
- Fang, T. & Zhang, J. 2009. Closed-form exact solutions of MHD viscous flow over a shrinking sheet. *Communications in Nonlinear Science and Numerical Simulation* 14: 2853-2857.
- Fang, T., Yao, S., Zhang, J. & Aziz, A. 2010. Viscous flow over a shrinking sheet with a second order slip flow model. *Communications in Nonlinear Science and Numerical Simulation* 15: 1831-1842.
- Fang, T., Liang, W. & Lee, C.F.F. 2008. A new solution branch for the Blasius equation - A shrinking sheet problem. *Computer & Mathematics with Applications* 56: 3088-3095.
- Hassanien, I.A. 1996. Flow and heat transfer on a continuous flat surface moving in a parallel free stream of power-law fluid. *Applied Mathematical Modelling* 20: 779-784.
- Hayat, T., Abbas, Z. & Sajid, M. 2007. On the analytic solution of magnetohydrodynamic flow of a second grade fluid over a shrinking sheet. *Journal of Applied Mechanics* 74: 1165-1171.
- Howell, T.G., Jeng, D.R. & De Witt, K.J. 1997. Momentum and heat transfer on a continuous moving surface in a power law fluid. *International Journal of Heat and Mass Transfer* 40: 1853-1861.
- Ishak, A., Lok, Y.Y. & Pop, I. 2010. Stagnation-point flow over a shrinking sheet in a micropolar fluid. *Chemical Engineering Communications* 197: 1417-1427.
- Mahapatra, T.R., Nandy, S.K. & Gupta, A.S. 2009. Analytical solution of magnetohydrodynamic stagnation-point flow of a power-law fluid towards a stretching surface. *Applied Mathematics and Computation* 215: 1696-1710.
- Miklavčič, M. & Wang, C.Y. 2006. Viscous flow due to a shrinking sheet. *Quarterly of Applied Mathematics* 64: 283-290.
- Postelnicu, A. & Pop, I. 2011. Falkner-Skan boundary layer flow of a power-law fluid past a stretching wedge. *Applied Mathematics and Computation* 217: 4359-4368.
- Prasad, K.V., Datti, P.S. & Vajravelu, K. 2010. Hydromagnetic flow and heat transfer of a non-Newtonian power law fluid over a vertical stretching sheet. *International Journal of Heat and Mass Transfer* 53: 879-888.
- Schowalter, W.R. 1960. The application of boundary-layer theory to power-law pseudoplastic fluids: Similar solutions. *AICHE Journal* 6: 24-28.
- Wang, T.Y. 1994. Similarity solution of laminar mixed convection heat transfer from a horizontal plate to power-law fluid. *Mingchi Institute of Technology Journal* 26: 25-32.
- Wilkinson, W.L. 1960. *Non-Newtonian Fluids*. London: Pergamon Press.
- Xu, H. & Liao, S.J. 2009. Laminar flow and heat transfer in the boundary-layer of non-Newtonian fluids over a stretching flat sheet. *Computer & Mathematics with Applications* 57: 1425-1431.
- Yao, S., Fang, T. & Zhong, Y. 2011. Heat transfer of a generalized stretching/shrinking wall problem with convective boundary conditions. *Communications in Nonlinear Science and Numerical Simulation* 16: 752-760.
- Zheng, L., Ting, L. & Zhang, X. 2008. Boundary layer flow on a moving surface in otherwise quiescent pseudo-plastic non-Newtonian fluids. *Metallurgy* 15: 241-244.

Nor Azizah Yacob  
Faculty of Computer and Mathematical Sciences  
Universiti Teknologi MARA Pahang  
26400 Bandar Tun Razak Jengka, Pahang  
Malaysia

Anuar Ishak\*  
School of Mathematical Sciences  
Faculty of Science and Technology  
Universiti Kebangsaan Malaysia  
43600 UKM Bangi, Selangor  
Malaysia

\*Corresponding author; email: anuar\_mi@ukm.my

Received: 24 August 2012

Accepted: 8 June 2013